Possible Symmetrization of Fisher's Droplet Picture near T_c : A Bubble–Droplet Formula

D. Stauffer,[1] C. S. Kiang,[2,3] and G. H. Walker[2]

An expression for the pressure is proposed which leads to a symmetric equation of state for liquid and gas near the critical point. Our "bubble–droplet" formula is similar to Fisher's cluster expansion but contains an additional term due to the density dependence of the surface tension. Also, it assumes the density difference between a droplet (or bubble) and the surrounding medium to be proportional to $l^{-1/\delta}$ and not to be independent of the droplet size l. Then, the scaling homogeneity assumption and some scaling laws, including $d\nu = 2 - \alpha$, can be derived (d is dimensionality). The additional assumption of spherical droplets and bubbles leads to a new scaling law $1 + \beta = (d - 1)\nu$, which is only slightly violated in the lattice gas for $d = 2, 3, 4$.

1. INTRODUCTION

Fisher's three-dimensional liquid droplet model[1,2] is a semiphenomenological description of a gas near the liquid–gas transition. It takes into account small liquid droplets in the gas phase below the critical temperature T_c if the chemical potential μ is smaller than some $\mu_c(T)$ analytic in temperature. For $\mu > \mu_c$, there is a divergence in the droplet model which is interpreted as the instability due to the formation of macroscopic liquid droplets. Thus, $\mu = \mu_c$ gives the coexistence curve where the gas condenses into the liquid. However, the droplet model cannot describe the liquid phase which exists for $\mu \geqslant \mu_c$ and $T < T_c$ and which behaves similarly to the gas

---

[1] Physik Department T30, Technische Universität, 8046 Garching, Reaktor, West Germany.
[2] Physics Department, Clark College, Atlanta, Georgia. Work partially supported by Research Corporation, a Frederick Gardner Cottrell grant aid.
[3] Address reprint requests to C.S.K.

phase very near T_c. This violation of the experimentally established symmetry between liquid and gas near the critical point is due to the fact that only droplets in the gas phase and no bubbles in the liquid phase are taken into account[1]. In this paper, we propose a formula which can be interpreted as taking into account both bubbles and droplets, and which fulfills this symmetry requirement without leading to unallowable singularities at the critical isotherm[2]. Our proposed formula was found mainly from mathematical considerations about how to fulfill the various requirements with expressions similar to Fisher's droplet model. We hope this publication will help others in finding a better physical justification than we are giving now. (Another symmetric expression was given by Fisher[3]; see also Ref. 4.)

We outline in the next section Fisher's droplet model[1], and in Section 3 our proposed bubble–droplet formula. Section 4 discusses, for general dimensionality d, the volume, the surface area, and the radius of the droplets and bubbles. Also discussed are the scaling law $d\nu = 2 - \alpha$ and a new relation $1 + \beta = (d - 1)\nu$ resulting from the assumption: surface $\propto$ (volume)$^{(d-1)/d}$. For Section 4, one does not have to assume the particular formula proposed in Section 3. In Section 5, various qualitative properties of our bubble-droplet formula are derived. We use the standard notation for the critical indices[1].

2. THE DROPLET PICTURE

Here, the equation of state for the simple droplet picture is made plausible.

In Fisher's asymmetric droplet model,[1] the free energy f_l of a droplet containing l molecules consists of a bulk term proportional to l, a surface term = (surface free energy) $\times$ (surface area) $\propto (T_c - T)\, l^\sigma$, and a positive logarithmic term $\tau\, kT \ln l$. All terms are analytic in temperature. Various theories for the logarithmic term have been proposed in connection with the nucleation of supersaturated vapors,[5] most of them not giving the desired positive value of τ ($=2 + 1/\delta = 2.2$). A positive contribution around $kT \ln l$ can be found from the fact that the wavelengths of excitations are restricted in a finite droplet.[6,14] The essential advantage of Fisher's droplet model is that this factor τ, the exponent σ for the surface area, and the microscopic surface tension are not determined from theories but introduced as free parameters to be fitted to the bulk properties like coexistence curve and critical isotherm ($\tau = 2 + 1/\delta$, $\sigma = 1/\delta\beta$). An application to the nucleation process is compared by Eggington et al.[7] with experiments near T_c.

The number n_l of droplets with l molecules is then assumed to be

$$\exp[-(f_l - \mu l)/kT],$$

and the pressure is $P = \sum_l n_l kT$ as a generalization of the ideal gas equation of the ideal gas equation $P = n_1 kT$ ($\rho = \partial P/\partial \mu = \sum_l n_l l$, as it should be). Thus, any droplet–droplet interaction is neglected. The coexistence curve is reached if μl, the chemical potential per droplet, equals the bulk term in f_l; for larger μ, the sum diverges: we have condensation. Thus, the pressure in Fisher's droplet picture is

$$P/kT = q_0 \sum_{l=1}^{\infty} l^{-\tau} x^{l^\sigma} y^l \tag{1}$$

with $\ln x = t/D$ ("surface"), $\ln y = \mu/kT -$ const ("bulk"), $\sigma = 1/\delta\beta < 1$, $\tau = 2 + 1/\delta$. Here, D and q_0 are constants (roughly, $D = 1/2$, $q_0 = \rho_c/5$), and $t = (T - T_c)/T_c$. The coexistence curve is given by $y = 1$; for $y > 1$, the sum diverges. The particle density ρ is $\partial(P/kT)/\partial(\ln y)$. In order to have $\rho = \rho_c$ above T_c ($x > 1$), we need[2] $\ln y(\rho_c) \propto t^{\delta\beta}$; this contradicts[2] experiment and scaling analyticity assumptions and means that we cannot use the formula for paramagnets above T_c ($y = e^{-H}$ in suitable units for the magnetic field H). Reatto[4] recently proposed a simple modification of (1) which avoids this nonanalytic behavior, may give $\rho(y = 1) = \rho_c$ above T_c, but is still not symmetric.

In addition, for spherical droplets in three dimensions, the surface exponent σ should be 2/3; for nonspherical ones, it may be greater but not smaller. However, this geometrical condition $\sigma = 1/\delta\beta \geqslant 2/3$ is violated in most cases. Finally, below T_c, only the gas phase, $\rho_c > \rho$, and not the liquid phase, $\rho > \rho_c$ can be described. Nevertheless, for $\rho_c > \rho$, the overall agreement with experiment is good.[2]

3. THE BUBBLE–DROPLET FORMULA

Here, heuristict arguments are given for an improved ansatz.

In order to have a more symmetric model, we not only need droplets but also bubbles. We assume that for bubbles and droplets of the same size, the surface terms are the same, whereas the bulk term changes its sign. An expression like $\sum l^{-\tau} x^{l^\sigma}(y^l + y^{-1})$, however, would nearly alwways diverge. But near the critical point, there will also exist bubbles within the droplets, and droplets within the bubbles within the droplets, etc. Perhaps this series has some similarity with a geometric series (see below). The expression $\sum l^{-\tau}[\text{const} + x^{l^\sigma}(y^l + y^{-1})]^{-1}$ indeed never diverges and is symmetric for liquid ($y \geqslant 1$) and gas ($y \leqslant 1$). However, this sum does not yield a coexistence curve, where for one x and y we have two different densities, ρ_L and ρ_G for liquid and gas, respectively. Also, an equation $\rho = f(x, y)$ can give two values for ρ if x or y depend on ρ. Indeed, there is a physical reason to introduce a ρ-dependent x: $-\ln x$ is a (microscopic) surface tension, and any surface tension should be the larger the greater the density difference between the two media separated by the surface. Thus, we might write near the critical point ($\rho \approx \rho_c$) for the high density droplets: Surface free energy per molecule $= -\ln x \propto [-t(\text{const} - \rho + \rho_c)]$; and correspondingly $-t(\text{const} + \rho - \rho_c)$ for the low-density bubbles. However, the term const $\pm (\rho - \rho_c)$ is not compatible with the desired scaling homogeneity near T_c.

In order to circumvent this difficulty, we apply an idea of Kadanoff's.[8] Until now, we assumed the droplets (bubbles) to have a *constant* density much greater smaller) than the surrounding density $\rho \approx \rho_c$. However, if the scaling law $dv = \gamma + 2\beta$ is valid, then the order-parameter fluctuations in a volume ξ^d (d is dimensionality, ξ is coherence length) have the same temperature dependence at the coexistence curve as the equilibrium order parameter (and the same field dependence at the critical isotherm). If we identify our bubbles and droplets with these small fluctuations around the average density ρ, then the difference between droplet (bubble) density and the surrounding density ρ should vanish if the critical point is reached. The typical droplet size is proportional to $(-t)^{-1/\delta}$ and $(\Delta\mu)^{-1}$ according to Eq. (1).

Thus, we choose $\rho_{\text{droplet}} - \rho \approx \rho - \rho_{\text{bubble}} \propto l^{-1/\delta}$ similarly to Kadanoff's proposal. More precisely, $\rho_{\text{droplet}} - \rho$ and $\rho - \rho_{\text{bubble}}$ should be the same if $\rho = \rho_c$. If, however, $\rho > \rho_c$, the, at least in a lattice gas $(0 \leqslant \rho \leqslant 2\rho_c)$, the maximum density fluctuation for a droplet $(2\rho_c - \rho)$ is smaller than for a bubble $(\rho - 0)$; thus, we also expect the typical density flucutuation to be smaller for a droplet than for a bubble. (The opposite should occur if $\rho < \rho_c$.) The (microscopic) surface tension is therefore assumed to be proportional to $t[l^{-1/\delta} \pm (\rho - \rho_c) \text{ const}]$, with plus for bubbles and minus for droplets. With a surface area proportional to l^σ., we thus have a surface free energy proportional to $[tl^{\sigma'-1/\delta} \pm tl^{\sigma'}(\Delta\rho) \text{ const}] = tl^\sigma \pm tl^{\sigma+1/\delta}(\Delta\rho) \text{ const}$. This expression is compatible with scaling homogeneity. Here, $\sigma = \sigma' - 1/\delta$ is no longer identical to the surface area exponent σ', but still given by $\sigma = 1/\delta\beta$ (cf. Reatto[4])

Using these assumptions, the result would be

$$P = P_0 - kTq_0 \sum_{l=1}^{\infty} l^{-\tau}(a + x^{l\sigma}y^l z^{l\sigma+1/\delta} + x^l y^{-l} z^{-l\sigma+1/\delta})^{-1} \tag{2a}$$

$$\ln x = t/D, \qquad \ln y = \Delta\mu, \qquad \ln z = -t\,\Delta\rho/E \tag{2b}$$

$$t = (T - T_c)/T_c, \qquad \Delta\rho = (\rho - \rho_c)/\rho_c, \qquad \Delta\mu = (\mu - \mu)_c/kT \tag{2c}$$

$$q_0 > 0, \qquad D > 0, \qquad E > 0, \qquad a > 2 \tag{2d}$$

Here, P_0, with $\partial P_0/\partial\mu = \rho_c$, is an analytic term not connected with bubbles and droplets. On the critical isochore above T_c and on the coexistence curve below T_c, y is equal to unity and $\mu = \mu_c$ is analytic in t. Contrary to (1), the sum (2) never diverges.

The main points unsolved in our "deverivation" are the contributions of bubbles within droplets, etc. These complicated configurations seem to be essential for the critical behavior because with

$$a_l = x^{l\sigma}y^l z^{l\sigma+1/\delta} + x^{l\sigma}y^{-l}z^{-l\sigma+1/\delta} \tag{2e}$$

the sum $\sum l^{-\tau}a_l$ diverges; it corresponds in our interpretation to single bubbles and droplets not being included in each other. We do not have a justification for our geometric series ansatz (2a): $\sum l^{-\tau}f(a_l)$ with $f(a_l) = (a + a_l)^{-1}$. Thus, we write our final proposal more generally as

$$P = P_0 - kTq_0 \sum_{l=1}^{\infty} l^{-\tau}f(a_l) \tag{2f}$$

with an unknown function f. Convergence is ensured if f and its derivatives are finite for $a_l \rightarrow 0$ and vanish for $a_l \rightarrow \infty$ not slower than some negative power of a_l. Also, this general function f is an approximation only because the bubble within a droplet may be larger or smaller than the droplet, however, is not esential phenomena because we will find agreement with scaling laws although we neglected this effect.

Also, in the phenomenological scaling analysis, one uses such unknown functions [for example, $\Delta\rho = t^\beta g(\Delta\mu/t^{\delta\beta})$]. But our function f does bot need to have the complicated asymptotic properties required for this scaling function g; we can give simple

examples for f fulfilling our requirements, whereas no closed expression for the scaling function g has yet been proposed. We hope that the detailed shape of our unknown function f is not very important.

4. SHAPE OF THE DROPLETS AND BUBBLES

Here, relations connecting volume, surface, and radius of a droplet (bubble) are discussed for general dimensionality.

Our assumption that the density within the droplets and bubles approaches ρ_c at the critical point differs from Fisher.[1] Therefore, the volume V_l of a bubble, where l molecules are missing compared to the average density, is not proportional to l, but to $l/$(density difference), which is proportional to $l^{1+1/\delta} \approx l^{1.2}$. The surface area S_l is proportional to $l^{\sigma'} = l^{\sigma+1/\delta} \approx l^{0.8}$. From geometrical considerations, we would like to have in d dimensions: $S_l V_l^{(1-d)/d} \propto l^b$, with $b \geqslant 0$. The additional assumption of spherical droplet (or $b = 0$, more generally) leads by $S_l \propto V_l^{(d-1)/d}$ to the new scaling law

$$1 + \beta = (d - 1)\nu \tag{3}$$

Experiments are in general not accurate enough to exclude reliably this relation. In the Ising model,[9] it is slightly violated for $d = 2$ ($1.125 = 1 + \beta > \nu = 1$) and nearly correct for $d = 3$ ($1.31 = 1 + \beta > 2\nu = 1.28$) and $d = 4$ ($1 + \beta \approx 1.5 \approx 3\nu$). For $d = 3, 4$, the deviations are of the same order of magnitude as the deviations from $d\nu = 2 - \alpha$, assumed for our analysis. (Mean field theories are excluded because of $d\nu \neq \gamma + 2\beta$; in the three-dimensional spherical model for isotropic ferromagnets, $1 + \beta = 1.5 < 2\nu = 2$, but our picture with narrow domain walls should be more accurate for strongly anisotropic magnets.) Thus, the droplets can be assumed to be nearly spherical for $d = 3$ and $d = 4$; the deviations for $d = 2, 3, 4$ from the scaling law (3) are in the direction to be expected for nonspherical droplets ($b > 0$). On the other hand, Fisher's droplet picture[1] gives $b = \sigma + (1 - d)/d < 0$ for $d = 3$. Thus, in general, we no longer need Fisher's not yet justified assumption that due to droplet–droplet interactions (excluded volume effects) the effective surface exponent is made smaller than the geometrical one for $d = 3$. Notice that for $1 + \beta = (d - 1)\nu$, the microscopic surface tension $t l^{-1/\delta}$ has for the typical droplets on the coexistence curve the same temperature dependence as the macroscopic one.[10]

Also, if the droplets are not spherical, the coherence length should be $\sim V^{1/d}$ for the typical droplet size because the density flucutations within a volume V_l were assumed to be proportional to the equilibrium order parameter. Thus, we have $\xi \propto l^{(1+1/\delta)/d} = l^{(2-\alpha)/\delta\beta d} = l^{\sigma\nu}$, as it should be. Generally, we interpret $l^{\sigma\nu}$ as the radius of a bubble or droplet. We can conclude for $d\nu = 2 - \alpha$, $d = 3$:

$$\begin{array}{ll} \text{radius} & \propto l^{\sigma\nu} \approx l^{0.4} \\ \text{surface} & l^{\sigma+1/\delta} \approx l^{0.8} \\ \text{volume} & l^{1+1/\delta} \approx l^{1.2} \end{array} \tag{4}$$

The scaling law $d\nu = \gamma + 2\beta$ is essentially assumed in our derivation of the

bubble–droplet picture and leads to the homogeneous scaling equation of state. Instead, one might first assume $\rho_{\mathrm{droplet}} - \rho \propto l^{-\psi}$, with arbitrary ψ. If, then, scaling homogeneity is assumed for the resulting bubble–droplet formula, then $\psi = 1/\delta$ and thus $dv = 2 - \alpha$ (if volume $\sim \xi^d$ for the typical bubbles and droplets).

For large dimensionality d, one expects mean field exponents. Thus $dv \neq 2 - \alpha$ for $d > 4$, in contradiction to our results. Also, for $d \to \infty$, the surface area should be proportional to the volume; but it is not in our formula. Why is our picture wrong for large dimensionality? One possible reason might be that all our singularities come from the bubbles and droplets and not from the molecules in between. This approximation is probably wrong for $d \to \infty$, because the volume of a unit sphere $\pi^{d/2}/(d/2)!$ shrinks rapidly for large d. Thus, for high dimensionality, only the contributions of a very small fraction of molecules are taken into account; this can hardly be a good approximation. According to this argument, already for $d = 3$ our results should be worse than for $d = 2$. And indeed, in the lattice gas, $dv = 2 - \alpha$ is exact for $d = 2$ and slightly violated for $d = 3$.

5. QUALITATIVE PROPERTIES OF THE BUBBLE–DROPLET FORMULA

Now we discuss some properties of our ansatz (2f).

Symmetry and Analyticity. The sum for the pressure is completely symmetric for liquid and gas: If $\Delta\rho \to -\Delta\rho$, $\Delta\mu \to -\Delta\mu$, then $x \to x$, $y \to 1/y$, $z \to 1/z$; thus, the sum remains unchanged. Because of $\partial P/\partial \mu = \rho$, this means that $\Delta\rho$ is an antisymmetric function of $\Delta\mu$, as it should be very near the critical point.

[The rectilinear diameter $\bar{\rho} = (\rho_L + \rho_G)/2$ equals ρ_c if we neglect the temperature dependence of the analytic part $P_0(\mu, T)$ in (2). This is correct in magnets or lattice gases; but for real fluids, $\bar{\rho} - \rho_c \propto t$ or $\propto t^{1-\alpha}$. We can generalize (2) by choosing $\partial P_0/\partial\mu = \rho_c + \mathrm{const} \cdot t + \cdots$ and $\ln z = -t(\rho - \partial P_0/\partial\mu)/E\rho_c$. Then, $\bar{\rho} = \partial P_0/\partial\mu$ is analytic in t; and above T_c, $\rho = \partial P_0/\partial\mu \neq \rho_c$ at zero "field" $\ln y$ (maximum of $\partial\mu/\partial\mu$ not at ρ_c). This finite slope of the diameter means a "skew axis"[1] for ρ. Now, above T_c, $\mu(\rho_c, T) - \mu(\rho_c, T_c) \propto t^{\gamma+1}$; thus, $\mu_c(t)$ is not analytic as in Refs. 1, 2, but $d^2\mu_c/dt^2$ is continuous at T_c, in agreement with experiment.[2] This result might be important independent of any droplet theories. From now on, we neglect this possible finite slope of rectilinear diameter $\bar{\rho}$ (cf. Stauffer et al.[15]).]

Away from the critical point, the pressure (2f) with the choices $f = 1/(a + a_l)$ or $f = \exp(-a_l/a)$ is probably analytic in t and $\Delta\mu$ apart from essential singularities for $\Delta\mu = 0$ both above and below T_c.

Homogeneity. The derivative with respect to μ gives [using $f' = df(a_l)/da_l$ with a_l from (2e)]

$$-\Delta\rho = \frac{q_0}{\rho_c} \sum_l l^{1-\tau} x^{l\sigma} f'(e_l - 1/e_l) \left[1 - l^{\sigma - 1 + 1/\delta} t \, \frac{\partial(\Delta\rho)}{\partial(\Delta\mu)} \Big/ E \right] \qquad (5a)$$

or, with $M^* = \Delta\rho/\Delta\mu^{1/\delta}$, $t^* = t/\Delta\mu^\sigma$, $v = l\,\Delta\mu$, $\Delta\mu > 0$:

$$-M^* = \frac{q_0}{\rho_c} \int_0^\infty v^{1-\tau}[\exp(t^* v^\sigma/D)]\, f'(e_l - 1/e_l)$$

$$\cdot \left[1 + v^{\sigma-1+1/\delta}t^* \left(\frac{\sigma t^*}{E}\frac{dM^*}{dt^*} - \frac{M^*}{E\delta}\right)\right] dv \tag{5b}$$

Here, we used the abbreviation $e_l = y^l z^{l^{\sigma+1/d}} = \exp(v - t^* v^{\sigma+1/d} M^*/E)$. Equation (5b) fulfills as well as (1), the scaling homogeneity assumption. It is a first-order differential equation $\partial\rho/\partial\mu = F(\rho, \mu, t)$. [The equation of state is roughly correct for $f = 1/(a + a_l)$, $a = 2, 5$, $q_0/\rho_c = 5$, $D = 1$, $E = 3/2$.]

Coexistence Curve. For $\Delta = \mu 0$ above T_c, there should be no solution $\Delta\rho \neq 0$ of (5a); below T_c for $\Delta\mu = 0$, we need, besides the trivial solution $\Delta\rho = 0$, another one with $\Delta\rho \neq 0$: the coexistence curve. We see that (5) might fulfill this requirement; for, if we replace l by its typical value $|t|^{-\delta\beta}$ for small $\Delta\mu$, replace f' by a negative constant, and neglect the correction term $\propto l^{\sigma-1+1/\delta}$, then (3a) looks like $\Delta\rho\,|t|^{-\beta} = Q[\exp(|t|^{-\delta\beta}\Delta\mu - t\,|t|^{-\beta-1}\Delta\rho/E) - \exp(-|t|^{-\delta\beta}\Delta\mu + t\,|t|^{-\beta-1}\Delta\mu/E)]$ with positive Q. Above T_c, the only solution for $\Delta\mu = 0$ is $\Delta\rho = 0$; below T_c, there is another solution if $Q < E/2$.

Critical Isotherm. At T_c, the critical isotherm $\Delta\rho = M^*\Delta\mu^{1/\delta}$, for $f = 1/(a + a_l)$, is given by

$$M^* = \frac{q_0}{\rho_c}\int_0^\infty v^{1-\tau}[(e^v - e^{-v})/(a + e^v + e^{-v})^2]\, dv = q_0/5\rho_c \tag{6}$$

for $a = 2.5$. The correction terms to this asymptotic $\Delta\mu \propto \Delta\rho^\delta$ law are known for the Ising model[11]: at $T = T_c$, $\Delta\rho \propto \Delta\mu^{1/\delta}(1 + \text{const}_1\,\Delta\mu^{1/\delta} + \text{const}_2\,\Delta\rho^{\delta-1} + \cdots)$. If we do not replace the sum (2) by the integral (6), we find a $\Delta\rho^{\delta-1}$ term arising from the missing $l = 0$ term. But no $\Delta\rho^{1/\delta}$ correction can be found because at $T = T_c$, only τ and not $\beta = (\tau - 2)/\sigma$ occurs. [This holds as well for Fisher's simple formula (1).] Thus, one should not use (1) or (2) except very near the critical point, where large droplets only are important.

Specific Heat. For $\alpha = 0$, the specific-heat singularities for $\Delta\mu = 0$ above and below T_c are proportional to the divergences in $d^2 P^\pm/dT^2$, where P^+ is the pressure on the critical isochore and P^- the vapor pressure. We find for the singular part the expected symmetric logarithmic peak; with the choice $f(a_l) = 1/(a + a_l)$, it is

$$C_{v,\text{sing}}^+ = C_{v,\text{sing}}^- = kq_0(\gamma + \beta)\,2D^{-2}(a - 2)(a + 2)^{-3}\, ln\,|t^{-1}| \tag{7}$$

The specific heat is infinite (for $\alpha \geqslant 0$) only at $t = 0$, $\Delta\rho = 0$. This is not obvious *a priori*; for, the specific heat might diverge below T_c in the interior of the coexistence curve if $\partial\rho/\partial\mu = \infty$ ("spinodal line"), because here the flucutations of the density diverge.

Compressible Spin Systems. According to Wagner,[12] the $x^{l^\sigma}y^l$ term in the droplet model should be multiplied by w^{l^λ} ($w < 1 =$ const), if thermal lattice vibrations are taken into account in a ferromagnet below T_c ($t < t_0$), this effect causes a rounding of the specific heat and of other singularities. An analogous correction to our bubble–droplet formula (2) would give similar rounding effects above and below T_c. But according to Wagner,[12] λ is the exponent for the droplet radius; in our picture (Section 4), this exponent is $\sigma\nu \approx 0.4$ instead of Wagner's $\lambda = \sigma/2 \approx 0.3$. Then, Wagner's t_0 for nickel ($=3 \sim 10^{-6}$) is enlarged by one order of magnitude.

6. DISCUSSION

We conclude that our bubble–droplet formula fulfills the homogeneity and symmetry requirements of the scaling laws and most of their analyticity assumptions. It also allows us to assume the droplets as (nearly) spherical, giving a new scaling law $1 + \beta = 2\nu$. Using this and the other scaling laws, we can determine all critical exponents from δ. For nonpolar fluids, δ can be calculated[13] by a semiempricical formula from the compressibility factor $P_c/\rho_c kT_c$. If the equals 0.27, we find $\alpha = \eta = 0$, $\beta = 1/3$, $\nu = 2/3$, $\gamma = 4/3$, $\delta = 5$. The disadvantage of our proposed formula is its complicated nature. Also we would like to have a more reliable physical justification for it, in particular for the function f.

ACKNOWLEDGMENTS

REFERENCES

1. M. E. Fisher, *Physics* **3**:255 (1967); M. E. Fisher and B. U. Felderhof, *Ann. Phys.* **58**:176, 217, 268, 281 (1970).
2. C. S. Kiang and D. Stauffer, *Z. Physik* **235**:130 (1970).
3. M. E. Fisher, private communication.
4. J. B. Jalickee *et al.*, preprint; L. Reatto, *Phys. Letters* **33A**:519 (1970).
5. J. L. Katz, *J. Stat. Phys.* **2**(2):137 (1970), and references therein.
6. F. H. Stillinger, Jr., *J. Chem. Phys.* **47**:2513 (1967).
7. A. Eggington, C. S. Kiang, D. Stauffer, and G. H. Walker, *Phys. Rev. Letters* **26**:820 (1971).
8. L. P. Kadanoff, Lecture Notes, International School of Physics "Enrico Fermi," LI course: Critical Phenomena, Varenna, 1970.
9. M. A. Moore, *Phys. Rev.* **B1**:2238 (1970), and references therein.
10. S. Fisk and B. Widom, *J. Chem. Phys.* **50**:3219 (1969); D. Stauffer, *Z. Physik* **221**:122 (1969).
11. C. Domb, talk at Varenna (Ref. 8).
12. H. Wagner, *Phys. Rev. Letters* **25**:31 (1970).
13. C. S. Kiang, *Phys. Rev. Letters* **24**:47 (1970).
14. C. S. Kiang, D. Stauffer, G. H. Walker, O. P. Puri, J. D. Wise, Jr., and E. M. Patterson, to be published.
15. D. Stauffer, C. S. Kiang, G. H. Walker, O. P. Puri, J. D. Wise, Jr., and E. M. Patterson, *Phys. Letters* **35A**:172 (1971).